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Generalized Valon Model for Double Parton Distributions

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Abstract We show how the double parton distributions may be obtained consistently from the many-body light-cone wave functions. We illustrate the method on the example of the pion with two Fock components. The procedure, by construction, satisfies the Gaunt–Stirling sum rules. The resulting single parton distributions of valence quarks and gluons are consistent with a phenomenological parametrization at a low scale.

Multiparton distributions were considered already in pre-QCD times [1] and in the early days of QCD [2–4]. Recently, the interest in these objects has been renewed (see, e.g., [5–11] and references therein), with expectations that the double parton distributions (dPDF's) of the nucleon may be accessible from the double parton scattering contribution in certain exclusive production channels at the LHC [12,13]. Whereas dPDF's are well defined objects, their explicit construction, or parametrization that can be used phenomenologically, is difficult to achieve in a way where all the formal constraints are satisfied. In particular, the frequently used product ansatz or its modifications is inconsistent with the basic requirements of the Gaunt–Stirling (GS) sum rules [14,15].

On the model side, there are only a few explicit calculations in the literature: the studies in the MIT bag model [16] or in the constituent quark model [17] implement relativity approximately, hence the support of the distributions extends outside of the physical region. These problems were mended in Refs. [18,19]. On the other hand, a simple *valon* model [20,21] applied to the nucleon in [22] leads to dPDF's satisfying all formal constraints and to reasonable single parton distributions (sPDF's) for the valence sector. In this talk we pursue this idea, extending the valon model to include higher Fock-state components, which allows us to study the gluon and sea content.

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For practical purposes, it would be highly desirable to have a simple working parametrization for dPDF's in analogy to parameterizations of sPDF's. The ultimate goal of our approach is to systematically construct dPDF's which on one side satisfy all formal constraints, and on the other side reproduce the known sPDF's.

In phenomenological applications one assumes for simplicity the transverse-longitudinal decoupling in the dPDF's, $\Gamma_{ij}(x_1, x_2, k_T) = D_{ij}(x_1, x_2)f(k_T)$, moreover, one frequently applies the ansatz $D_{ij}(x_1, x_2) = D_i(x_1)D_j(x_2)\theta(1-x_1-x_2)(1-x_1-x_2)^n/(1-x_1)^{n_1}/(1-x_2)^{n_2}$, where $D_i(x)$ are the sPDF's. However, this form violates the GS sum rules [14]:

$$\int_0^{1-y} dx D_{i_{\text{val}}j}(x, y) = \int_0^{1-y} dx [D_{ij}(x, y) - D_{\bar{i}j}(x, y)] = (N_{i_{\text{val}}} - \delta_{ij} + \delta_{\bar{i}j})D_j(y), \quad (1)$$

$$\sum_i \int_0^{1-y} dx x D_{ij}(x, y) = (1-y)D_j(y), \quad (\text{and similarly for the second parton}).$$

This violation is formally a serious problem, as sum rules (1) follow from very basic field-theoretic features, namely the Fock-space decomposition of the hadron wave function and conservation laws. An attempt of a construction of the ansatz made in Ref. [23] met problems with the parton exchange symmetry and positivity. For the gluon sector of the proton, a formally successful ansatz has been obtained with the help of the Mellin moments [24].

The first issue we wish to point out in this talk is the non-uniqueness of the sPDF constraints. In other words, the sPDF's do not fix the dPDF's unambiguously. Suppose we have found a form of dPDF's which satisfies the sum rules (1). A sample function illustrating the situation follows from the valon model for the nucleon [22] with a single valence Fock component $|p\rangle = |uud\rangle$. It is presented in the left panel of Fig. 1. Projections lead to $D_u(x) = 2D_d(x) = 40x(1-x)^3$, where these valence sPDF's satisfy the GS sum rules, as can be explicitly seen:

$$D_u(y) = \int dx D_{du}(x, y) = \int dx D_{uu}(x, y), \quad 2D_d(y) = \int dx D_{ud}(x, y), \quad (2)$$

$$(1-y)D_u(y) = \int dx x [D_{du}(x, y) + D_{uu}(x, y)], \quad (1-y)D_d(y) = \int dx x D_{ud}(x, y).$$

To show that the solution for dPDF's leading to specified sPDF's is not unique, we may perturb the dPDF's by adding or subtracting strength in specific points, as shown in the right panel of Fig. 1. Then, by our specific choice of adding delta functions of strength ϵ and subtracting -2ϵ (or their multiplicities) at coordinates which are in proportion 2 : 5 : 8 does not contribute to the right-hand sides of the sum rule. One may smear this

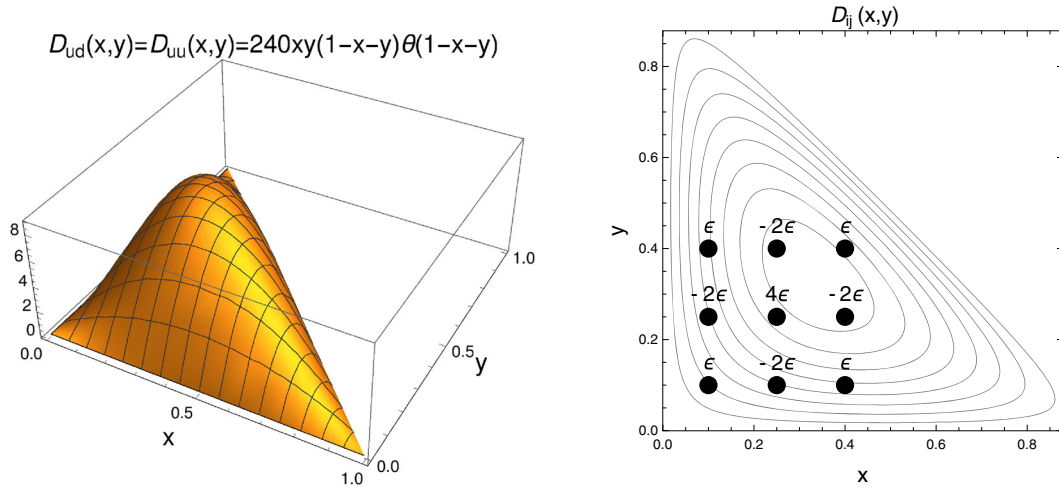


Fig. 1 *Left* example of the valence dPDF of the proton which satisfies the GS sum rules. *Right* schematic illustration of the non-uniqueness of the sPDF constraints

construction by using a distribution of points. From a general point of view, the non-uniqueness is obvious: one-particle distributions do not fix the two-particle distribution, which may contain correlations [25]. This shows that the “bottom-up” attempts to guess dPDF’s with just the constraints from sPDF’s are arbitrary.

As a remedy, we propose the “top-down” method which starts from the multiparticle light-cone wave function, extending the analysis of [22]. The approach guarantees the fulfillment of the formal requirement, in particular the GS sum rules (1) [15]. The Fock expansion of a hadron state in partonic constituents is assumed to have the form

$$|h\rangle = \sum_N \sum_{f_1 \dots f_N} \int dx_1 \dots dx_N \delta(1 - \sum_{k=1}^N x_k) \Psi_N(x_1 \dots x_N; f_1 \dots f_N) |x_1 \dots x_N; f_1 \dots f_N\rangle, \quad (3)$$

with f_i denoting the parton type. One should then model the N -parton Fock components Ψ_N ’s, obeying the constraints from the known sPDF’s. Having this, one may compute the double distributions, $D_{f_1 f_2}(x_1, x_2)$, and also the higher-particle distributions if needed.

We emphasize that the presented analysis based on Eq. (3) is effectively one-dimensional, with the transverse degrees of freedom integrated out. Non perturbative transverse lattice calculations suggest that such a dimensional reduction is triggered at lower renormalization scales set by the lattice spacing, $Q \sim 1/a_\perp$. Then, the transverse degrees of freedom are effectively frozen when the transverse momenta of the quarks are smaller than the lattice spacing [26,27] (see also the discussion in Ref. [22]).

We make the simplifying assumption that the only correlations in the wave function come from the longitudinal momentum conservation: $1 = x_1 + x_2 + \dots + x_N$ (the *generalized valon model*). Then

$$|\psi_N(x_1 \dots x_N; f_1 \dots f_N)|^2 = A^2 \phi_{f_1}(x_1) \dots \phi_{f_N}(x_N), \quad (4)$$

where $\phi_{f_1}(x) = |\psi_{f_1}(x)|^2$ and $\psi_{f_i}(x)$ is the one-body wave function of parton f_i , in principle computable in a dynamical model. Let the asymptotics of the single-parton functions be

$$\phi_{f_i}(x) \sim x^{\alpha_{f_i}-1} \quad \text{at } x \rightarrow 0, \quad \phi_{f_i}(x) \sim (1-x)^{\beta_{f_i}} \quad \text{at } x \rightarrow 1, \quad (5)$$

where for integrability $\alpha_{f_i} > 0$ and $\beta_{f_i} > -1$. For $N = 2$ with parton types f and f' the resulting asymptotics for sPDF’s at $x \rightarrow 0$ and $x \rightarrow 1$ is $D_f(x) \sim x^{\alpha_f + \beta_{f'} - 1}$ and $D_{f'}(x) \sim (1-x)^{\beta_f + \alpha_{f'} - 1}$, whereas dPDF’s assume the singular form $D_{ff'}(x, y) \sim \phi_f(x) \phi_{f'}(y) \delta(1-x-y)$. The emergence of the singular part follows from the presence of just two partons. It would be washed out by the QCD evolution to higher scales [22], which redistributes the strength to higher Fock components. For $N \geq 3$

$$D_f(x) \sim x^{\alpha_f - 1} \quad \text{at } x \rightarrow 0, \quad D_f(x) \sim (1-x)^{\beta_f + \alpha_{f_1} + \dots + \alpha_{f_{N-1}} - 1} \quad \text{at } x \rightarrow 1, \quad (6)$$

where (...)’ indicates that index f is omitted in the sequence. We note that the $x \rightarrow 1$ behavior is sensitive to the low- x behavior of the other components, as in this limit the kinematics “pushes” them towards 0. The above formulas are useful in modeling, as they allow for matching of the asymptotic behavior of the single-particle functions ϕ_{f_i} to the phenomenologically known asymptotics of sPDF’s.

Before we apply the extended valon model to a specific case, let us make some remarks on the choice of the scale and the QCD evolution. With an increasing scale Q_0 , more and more partons at low x are generated, hence more and more Fock components are needed in the decomposition (3). For practical reasons of not dealing with too many components, it is then favorable to use the parameterizations for sPDF’s at a lowest possible Q_0 , such as, e.g., the GRV $_\pi$ [28] parametrization for the pion which uses Q_0 as low as 500 MeV. The tempting evolution to lower scales cannot be carried out too far down, as negative distributions are generated [29,30], as well as non-perturbative domain is entered. We also remark that the constituent quark models which do not have gluons or quarks at the *quark model scale* [31,32] do not generate sufficiently many gluons and sea quarks at higher experimental scales.

Explicitly, the GRV $_\pi$ [28] parametrization for the valence, gluon and sea sPDF’s reads

$$\begin{aligned} x V(x) &= 0.52 (0.38\sqrt{x} + 1) (1-x)^{0.37} x^{0.50}, \\ x g(x) &= (0.34\sqrt{x} + 0.68) (1-x)^{0.39} x^{0.48}, \quad x q_{\text{sea}}(x) = 0, \quad (Q_0 = 500 \text{ MeV}). \end{aligned} \quad (7)$$

In the following we will use the momentum fractions as constraints:

$$2 \int dx x V(x) = 0.58, \quad \int dx x g(x) = 0.42. \quad (8)$$

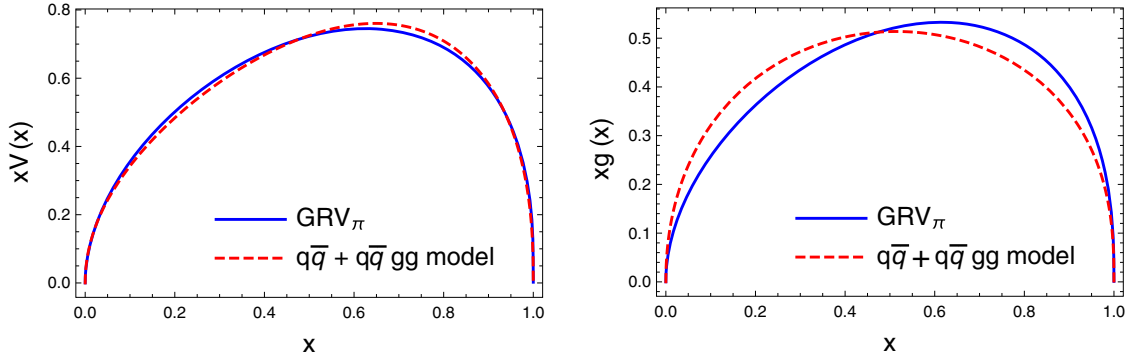


Fig. 2 The valence quark (*left*) and gluon (*right*) sPDF's of the pion from the two-component valon model (9, 12) compared to the GRV_π parametrization [28]

The average number of gluons inferred from Eq. (7) is $\int dx g(x) = 1.46$, hence a component with at least two gluons is necessary in the wave function.

We use the simple ansatz with just two Fock components:

$$|\pi^+\rangle = A|\bar{u}d\rangle + B|\bar{u}dgg\rangle. \quad (9)$$

Of course, this is a simplification, as we could have components with a single gluon and more than two gluons, as well as quark sea contributions. The corresponding sPDF's are:

$$D_{\bar{u}}(x) = A^2 |\Psi_{\bar{u}d}(x, 1-x)|^2 + B^2 \int dx_3 dx_4 |\Psi_{\bar{u}dgg}(x, 1-x-x_3-x_4, x_3, x_4)|^2 = D_d(x), \quad (10)$$

$$D_g(x) = B^2 \int dx_1 dx_2 (|\Psi_{\bar{u}dgg}(x_1, x_2, x, 1-x-x_1-x_2)|^2 + |\Psi_{\bar{u}dgg}(x_1, x_2, 1-x-x_1-x_2, x)|^2).$$

The conditions

$$A^2 + B^2 = 1, \quad (11)$$

$$\int dx x [D_{\bar{u}}(x) + D_d(x)] = 0.58 \quad \text{or} \quad \int dx x D_g(x) = 0.42$$

provide constraints for parameters. We use the generalized valon ansatz

$$|\Psi_{\bar{u}d}(x_1, x_2)|^2 \sim f(x_1; a, b) f(x_2; a, b), \quad (12)$$

$$|\Psi_{\bar{u}dgg}(x_1, x_2, x_3, x_4)|^2 \sim f(x_1; \alpha_q, \beta_q) f(x_2; \alpha_q, \beta_q) f(x_3; \alpha_g, \beta_g) f(x_4; \alpha_g, \beta_g)$$

with $f(x; \alpha, \beta) = x^{\alpha-1} (1-x)^\beta$. The choice $a+b=0.5$, $\alpha_q=0.5$, $\beta_q=-0.09$, $\alpha_g=0.48$, $\beta_g=-0.09$ is consistent with the asymptotic limit (6). Application of (11) yields $A^2=0.15$ and $B^2=0.85$, which means a strong dominance of the component with gluons over the $q\bar{q}$ component in the considered model.

The resulting sPDF's are shown in Fig. 2. We note a quite good (taking into account the simplicity of the model) agreement with the GRV_π parametrization, especially for the valence quarks. We can now tackle with our main task, namely, the determination of dPDF's. As already mentioned, the $N=2$ component generates a singular part $D_{\bar{u}d} \sim f(x; a, b) f(y; a, b) \delta(1-x-y) = D_{\bar{u}d}^{\text{sing}}(x, y) \delta(1-x-y)$. The $N=4$ component leads, upon integration over x_3 and x_4 , to broadly distributed functions. The result is shown in Fig. 3, proving that the proposed construction is practical.

An analogous construction for the case of the nucleon would be more challenging, as it requires at least three components in the Fock-space decomposition:

$$|p\rangle = A_{uud}|uud\rangle + \dots + A_{uud\bar{q}q}|uud\bar{q}q\rangle + \dots + A_{uudgg}|uudgg\rangle \quad (13)$$

This study, which would generalize the results for the valon model $|p\rangle = |uud\rangle$ considered in [22], is left for the future.

In conclusion, here are our main points:

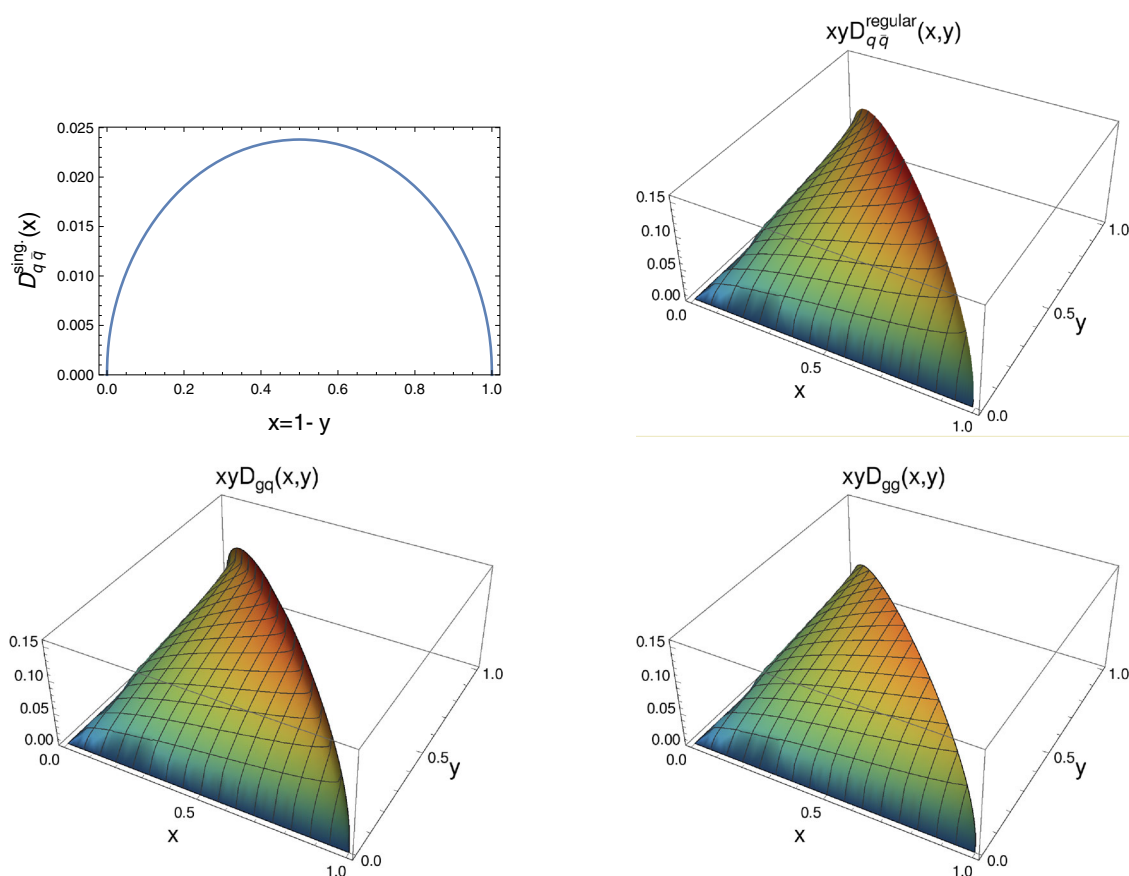


Fig. 3 dPDF's of the pion from the the two-component valon model (9, 12)

- The top-down strategy of constructing multi-parton distributions, which guarantees the formal features, is practical when the number of the Fock components is not too large. This is the case of dynamics at the lowest possible scale.
- The approach requires modeling of the light-cone wave functions, hence has physical input.
- Phenomenological sPDF's are used as constraints, but sPDF's alone cannot uniquely fix dPDF's.
- Many Fock components are needed to accurately reproduce the popular parameterizations of sPDF's, even at relatively low scales. The QCD evolution takes care of the generation of the higher Fock components at higher momentum scales.
- The valon model offers a simple ansatz at the initial low-energy scale that grasps the essential features with just the longitudinal momentum conservation, and the transverse degrees of freedom separated out from the dynamics.
- We note that the QCD evolution washes out the correlations in dPDF's at low x_1 and x_2 , justifying the approximate validity of the product ansatz in that limit. However, outside of that region the correlations are substantial.

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